BASIC CALCULUS

CHAPTER 2

THE 2-DIMENSIONAL COORDINATE SYSTEM

The graphs of linear equations in two variables are straight lines. Linear equations can be written in numerous ways:

Slope-Intercept Form: $y = mx+b$

In the equation $y=mx+b$, such as $y=-2x-3$, the slope is m and the y-intercept is (0,b). To graph equations of this type, use a table of values (Method 1) or the slope and y-intercept (Method 3).

General Form: $ax+by=c$

To graph equations like $3x-2y=-6$, locate the x- and y-intercepts (Method 2). Alternatively, solve the equation for y to write it as $y=mx+b$ and create a table of numbers.

Horizontal lines, $y=b$

The graph of $y=b$ is a horizontal line that passes through the point (0, b) on the y-axis. To graph an equation of this type, such as $y=4$, locate the point (0, b) on the y-axis and draw a horizontal line through it. If the equation does not have the form $y=b$, solve for y.

Vertical lines: $x = a$

The graph of $x=a$ is a vertical line that goes through the point (a,0) on the x axis. To graph a vertical line, such as $4x+12=0$, solve the equation for x and write it in the form $x=a$, then plot the point (a,0) on the x-axis and draw a vertical line through it.

METHOD 1: CONSTRUCT A TABLE OF VALUES

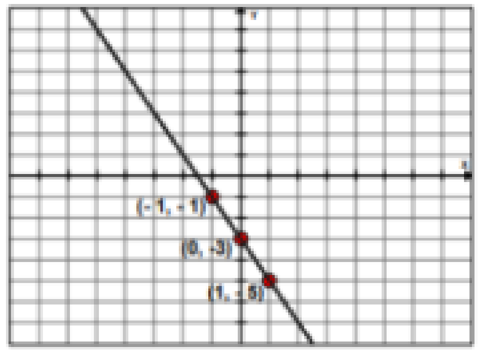
To graph equations of the form $y=mx$ or $y=mx+b$,

1) Select three values for x. Substitute these values into the equation and solve for the relevant y-coordinates.
2) Plot the ordered pairs identified in step 1.
3) Draw a straight line between the plotted spots. If the points do not line up, an error has occurred.

Example 1: Graph $y = -2x - 3$

Select three values for x and put them in a table to graph the equation. To determine the equivalent y-coordinate, substitute each value in the equation and simplify (Hint: pick easy-to-calculate values, such as −1, 0, and 1). Draw a straight line through each point on the plot of the ordered pairs.

x	y = −2x − 3	(x, y)
−1	$y = -2(-1) - 3$ $= 2 - 3 = -1$	(−1, −1)
0	$y = -2(0) - 3 = -3$	(0, −3)
1	$y = -2(1) - 3$ $= -2 - 3 = -5$	(1, −5)



Example 2: Graph 3x − 2y = −6

The equation 3x−2y=−6 is written in the general form. To graph this equation with a table of values, first solve the equation for y to write it in the form y=mx+b, as shown:

$$3x - 2y = -6$$

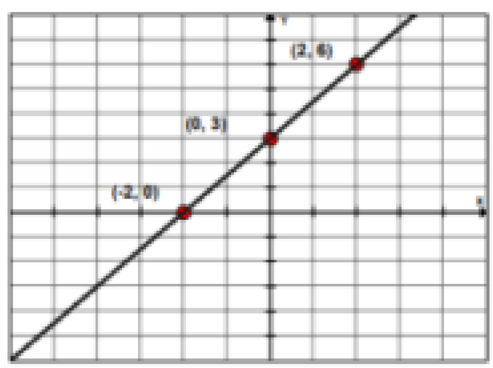
$$3x - 2y - 3x = -3x - 6$$

$$-2y = -3x - 6$$

$$y = \frac{3}{2}x + 3$$

Next, choose three values for x and calculate the corresponding y-coordinates. (Hint: to cancel fractions, choose multiples of the denominators.) Plot the points in the table and draw a line through them.

x	$y = \frac{3}{2}x + 3$	(x, y)
−2	$y = \frac{3}{\cancel{2}}(-\cancel{2}) + 3$ $= -3 + 3 = 0$	(−2, 0)
0	$y = \frac{3}{2}(0) + 3 = 0$	(0, 3)
2	$y = \frac{3}{\cancel{2}}(\cancel{2}) + 3$ $= 3 + 3 = 6$	(1, 6)



METHOD 2: FIND THE X AND Y-INTERCEPTS

1) To find the x-intercept, let y=0 then substitute 0 for y in the equation and solve for x.

2) To find the y-intercept, let x=0 then substitute 0 for x in the equation and solve for y.

3) Plot the intercepts, label each point, and draw a straight line through these points.

In Example 2, the line crosses the x-axis at $(-2,0)$ and the y-axis at $(0,3)$. The point at which the line crosses the x-axis is known as the x-intercept. At this point, the y-coordinate is 0. The point at which the line crosses the y-axis is known as the y-intercept. At this point, the x-coordinate is 0. When an equation is written in a generic form, such $-2x+4y=8$, it is easier to graph by identifying the intercepts.

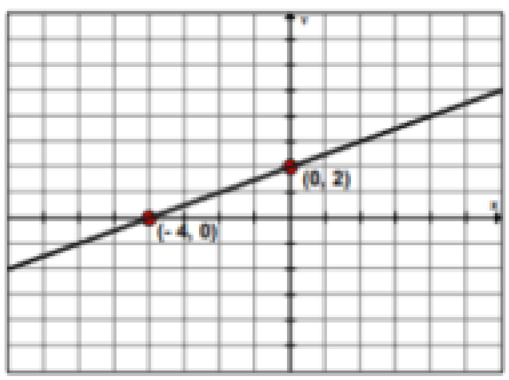
Example 3: Graph $-2x + 4y = 8$

1) To graph the equation, find the x and y-intercepts.

To find the x-intercept, let y=0 and solve To find the y-intercept, let x=0 and solve

the equation for x.	the equation for y.
$y = 0, -2x + 4y = 8$	$x = 0, -2x + 4y = 8$
$-2x + 4(0) = 8$	$-2(0) + 4y = 8$
$-2x = 8$	$4y = 8$
$x = -4$	$y = 2$
The x-intercept is (-4, 0)	The y-intercept is (0, 2)

2) Next, plot each intercept, label the points, and draw a line through them.



Graphing Horizontal and Vertical Lines

The graph of y=b is a horizontal line passing through the point (0, b), they intercept. The graph of x=a is a vertical line passing through the point (a,0), the x intercept.

Example 5: Graph $4x + 12 = 0$

First, solve the equation for x to write it in the form x=a.

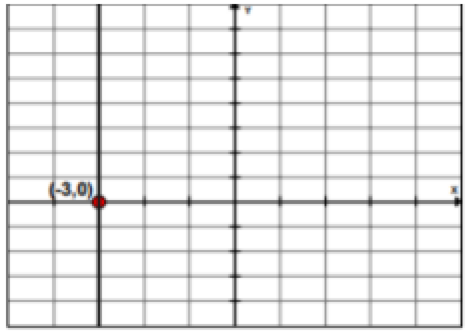
$$4x + 12 = 0$$

$$4x+= -12$$

$$x = -3$$

The x-intercept is $(-3,0)$. Plot this point on the x-axis, label the point, and draw a vertical

line through the point.



METHOD 3: USE THE SLOPE AND Y-INTERCEPT

To graph an equation, use the slope and y-intercept.

1) To find the slope m and y-intercept (0, b), write the equation as y=mx+b.
2) Now plot the y-intercept.
3) Move up, down, and left or right from the y-intercept, depending on whether the slope is positive or negative. Draw a point, then move up or down, left or right, to find a third point.
4) Draw a straight line connecting all three spots.

Example 6: Graph 2x + 5y = 10

To graph the equation using the slope and y-intercept, write the equation in the form y=mx+b to find the slope m and the y-intercept (0,b).

$$2x + 5y = 10$$

$$2x + 5y - 2x = -2x + 10$$

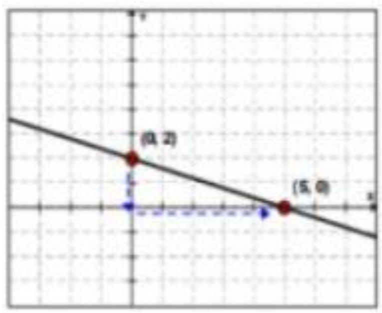
$$5y = -2x + 10$$

$$y = -\frac{1}{2}x + 2$$

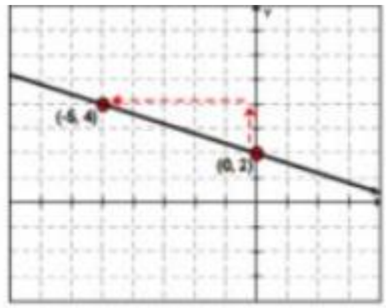
The slope m = $\frac{the\ change\ in\ y}{the\ change\ in\ x} = \frac{rise}{run} = -\frac{2}{5}$ and the y-intercept is (0, 2).

Now, plot the y-intercept. From there, move up or down two units (the rise) then move right or left five units to the right (the run) to find additional points.

When the slope is negative, make the change in y negative to locate points to the right of the y-intercept; make the ch ange in x negative to locate points to the left of the y intercept.



$m = \frac{-2}{5}$



$m = \frac{2}{-6}$

2.2. GRAPH OF INEQUALITIES

Graphing inequalities is similar to graphing equations except for a few minor differences

• If the graph consists of > or <, the line is dotted.

• If the graph consists of ≥ or ≤, the line is solid.

The graphing of the actual lines for inequalities is the same when graphing equalities.

Using the Symbols to Decide Where to Shade

1. Make sure the line(s) is in one of the following forms

a. $y > mx + b$

b. $y \geq mx + b$

c. $y < mx + b$

d. $y \leq mx + b$

2. Graph the lines as equations $y = mx + b$ but keep in mind if the line(s) need to be dotted or not.

3. Decide where to shade

a. If you have $y > mx + b$ or $y \geq mx + b$, then shade above the line.

b. If you have $y < mx + b$ or $y \leq mx + b$, then shade below the line.

4. When a problem asks for you to graph the solution set of a system of inequalities, start by going one line at a time. Graph the line and shade the correct side. Then repeat the process for the next line, until you have graphed and shaded for each of the lines given. Then look for the area that was shaded by all the inequalities. This area you will keep

shaded. For the other sections that are only shaded by one of the inequalities you erase the shading.

Note: For vertical inequalities of the form

a. $x > a$ or $x \geq a$, shade to the right

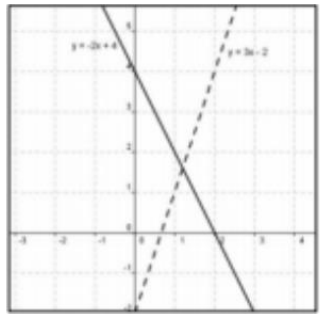
b. $x < a$ or $x \leq a$, shade to the left

Example: Graph the s o luti on se t of $y < 3x - 2$; $y \geq -2x + 4$

First we graph

y = 3x-2, this will have a dotted line

y = -2x + 4, this will have a solid line



Now we need t start shading

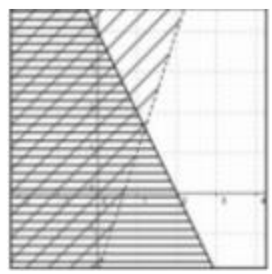
y < 3x-2, we shade below

y ≥ -2x + 4, we shade above

We will use

y<3x+2

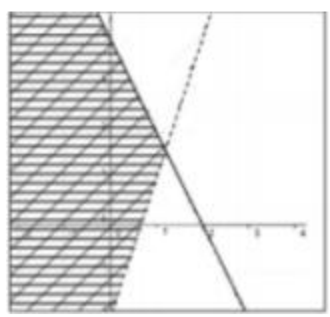
y≥-2x+4



Now we look for the area that has the double shade. This is the area we will keep. All the other shading we will erase. We now have graphed the solution set.

y < 3x-2, we shade below

y ≥ -2x + 4, we shade above



Using Test Point to Decide Where to Shade

Graphing with test points is helpful when you cannot remember what symbol means shade above or below. To use test points, we

1. Graph the inequality(s) (the same way as above)
2. Pick a point in a region that is not on any line
3. Put that point in to the inequality if the statement is true, shade the region that includes

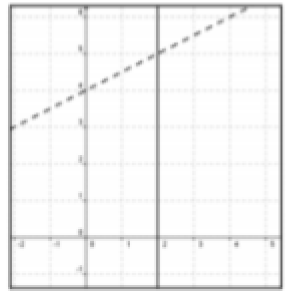
that point. If you have a system of inequalities, put the point into both inequalities. Both equations must be true for you to be able to shade.

4. Check other regions. These regions should make the inequality false.

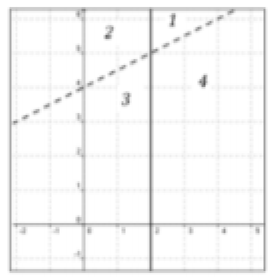
Example: Graph the solution set of $y < \frac{1}{2}x + 4$; $x \geq 2$

First Graph the lines

$y < \frac{1}{2}x + 4$ as a dotted line
$x \geq 2$ as solid line



From the graph, we can see that are four regions, which we have labeled for demonstrative purposes.



We will start with region 3. It doesn't matter what region you start with. Picking the point (0,0), which is the easiest point if you can do it. Plugging the point into the inequalities we get

$0 < \frac{1}{2}(0) + 4$ or $0 < 4$ which is true

$0 \geq 2$, which is false

Since both statements are not true, we cannot shade. Next, we'll try region 1. We pick the point (3,6) and plug this information into our inequalities we get

$6 < \frac{1}{2}(3) + 4$ or $6 < 5.5$ which is false

$3 > 2$, which is true

Since both statements are not true, we cannot shade.

REFERENCES

Winston S. Sirug. (2014). College Algebra. Revised Edition.

Louis Leithold. The Calculus With Analytic Geometry. Sixth Edition.